

On the Dark Sector Interactions

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Abstract

It is possible that there exist some interactions between dark energy (DE) and dark matter (DM), and a suitable interaction can alleviate the coincidence problem. Several phenomenological interacting forms are proposed and are fitted with observations in the literature. In this paper we investigate the possible interaction in a way independent of specific interacting forms by use of observational data (SNe, BAO, CMB and Hubble parameter). We divide the whole range of redshift into a few bins and set the interacting term $\delta(z)$ to be a constant in each redshift bin. We consider four parameterizations of the equation of state w_{de} for DE and find that $\delta(z)$ is likely to cross the non-interacting ($\delta = 0$) and have an oscillation form. It suggests that to study the interaction between DE and DM, more general phenomenological forms of the interacting term should be considered.

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I. INTRODUCTION

It has been suggested from astronomical observations that the main components of our universe are dark matter (DM) and dark energy (DE). DM behaves like the usual baryon matter and can form clusters, while DE is uniformly distributed in the whole universe, and it derives the universe to accelerating expand. Very ironically, we have known a little on DM and DE so far. The questions of what particles DM's are and what nature DE is remain open. From astronomical observations, however, some properties of DE can be deduced. For example, usually one characterizes DE with its equation of state w_{de} , the ratio of the pressure to the energy density of DE; w_{de} is found very close to -1 from the observations. Therefore a natural candidate of DE is the well-known cosmological constant introduced by Einstein in 1917, for which the equation of state is exact -1 . Although the cosmological constant is a beautiful and economic candidate, it suffers from some theoretical puzzles to be explained as currently observed DE. The theoretical difficulties (puzzles) are so-called fine-tuning problem and coincident problem (i.e., why energy densities of DE and DM happen to be of the same order today?)

To avoid these problems, some dynamical DE models have also been proposed in the literature. The simplest dynamical DE model is a time-dependent scalar field. Based on different forms of the Lagrangian of scalar field, the scalar field models could be classified into quintessence, K-essence, phantom and quintom models. Furthermore, due to the ignorance for DM and DE, one is not sure whether there exists any direct interaction between DM and DE, at least no known symmetries prevent such interaction. Indeed, possible interactions between DM and DE have been intensively investigated in recent years. It has been shown that a suitable interaction can help to alleviate the coincidence problem [1–4]. Various interacting models have been studied [5–15]. Several phenomenological interacting forms have been proposed and have been fitted with observations [16–22]. Some recent discussions seemingly imply that the decaying of DM into DE is favored [23, 24] by observations, which can make the coincidence problem more severe. However, most of those studies depend on the interacting forms, that is to say, those results are obtained by taking some special interacting terms. In other words, those studies are model dependent. Moreover, most of the models exclude the possibility of an oscillation interaction. And if the interaction exists, by fitting, one could only conclude that either DM decays to DE, or DE decays to DM.

In this paper, we are going to study the interaction in a way independent of the interacting form by observational data. To do that, we divide the whole redshift range into a few bins and the interacting term $\delta(z)$ is set to be a constant in each bin [25, 27]. Clearly such study depends on DE models and the number of bins. We will study 3-6 bins cases with a constant w_{de} and try to get some common features of $\delta(z)$. To see effect for different DE models, we will adopt four different parameterizations of w_{de} with a preferable division of bins. We will fit the interacting models with the Union SnIa [28], BAO [29], 9 Hubble data [30] and the shift parameter R from WMAP5 [31]. We obtain the best-fitted parameters and likelihoods by using the MCMC method. We find that $\delta(z)$ is likely to be oscillating and to cross the non-interacting ($\delta = 0$) line. We also compare behaviors of $r = \rho_m/\rho_{de}$ in the best-fitted models with those of corresponding models without interaction. In three cases of four parameterizations of w_{de} , the coincidence problem is alleviated, though DM decays into DE in some regions of redshift.

II. METHODOLOGY

We consider interacting models in a flat FRW universe

$$3H^2 = \rho_\gamma + \rho_b + \rho_{de} + \rho_{dm}, \quad (1)$$

where ρ_γ and ρ_b are energy densities of radiation and baryon, respectively, and ρ_{de} and ρ_{dm} are energy densities of DE and DM, respectively. We have set the speed of light $c = 1$ and $8\pi G = 1$. The continuity equations for energy densities of the interacting DM and DE are

$$\begin{aligned} \dot{\rho}_{dm} + 3H\rho_{dm} &= 3H\delta, \\ \dot{\rho}_{de} + 3H(1 + w_{de})\rho_{de} &= -3H\delta. \end{aligned} \quad (2)$$

In some phenomenological models of interaction, the interacting term δ is always assumed to be a function of ρ_{dm} and ρ_{de} , such as $\delta = \lambda\rho_{dm}$ [20, 21], $\delta = \lambda\rho_{de}$ [24, 32] or $\delta = \lambda(\rho_{dm} + \rho_{de})$ [2], thus the constraints resulting from observations will depend on the form of δ . By fitting, if $\lambda = 0$, it indicates that there does not exist interaction between DM and DE; if $\lambda > 0$, it stands for the decay direction from DE to DM; while from DM to DE, if $\lambda < 0$. However, obviously the way loses the possibility that δ has an oscillating behavior.

To investigate such a possibility, in this paper we divide the whole redshift into four bins and set δ to be a piecewise constant in each redshift bin

$$\delta(z_{n-1} < z \leq z_n) = \delta_n, \quad (n \geq 1) \quad (3)$$

In our main analysis, we will set $z_0 = 0$, $z_1 = 0.2$, $z_2 = 0.5$, $z_3 = 1.8$ and $z_4 = 1090$. Also we will consider possible effect of the number of bins on the fitting results.

From Eq. (2) we have

$$\begin{aligned} \rho_{dm}(z) &= \rho_{dm}^0(1+z)^3 - (1+z)^3 \int_0^z \frac{3\delta(x)}{(1+x)^4} dx, \\ \rho_{de}(z) &= \rho_{de}^0 F(z) + F(z) \int_0^z \frac{3\delta(x)}{(1+x)F(x)} dx, \end{aligned} \quad (4)$$

where $F(z) = \exp[\int_0^z \frac{3(1+w_{de})}{1+x} dx]$ and superscript 0 represents the present value. As $\rho_b = \rho_b^0(1+z)^3$, we can write ρ_{dm} and ρ_b together as ρ_m . With the piecewise constant $\delta(z)$ we have an analytical form for ρ_m

$$\rho_m(z_{n-1} < z \leq z_n) = [\rho_m^0 - \delta_1 + \sum_{i=1}^{n-1} (\delta_i - \delta_{i+1})(1+z_i)^3](1+z)^3 + \delta_n \quad (5)$$

where $\rho_m^0 = \rho_b^0 + \rho_{dm}^0$.

For energy density ρ_{de} of DE, we will employ four different parameterizations of w_{de} as follows.

I. $w_{de} = -1$. In that case, ρ_{de} can be written analytically as

$$\rho_{de}(z_{n-1} < z \leq z_n) = \rho_{de}^0 + 3 \sum_{i=1}^{n-1} (\delta_i - \delta_{i+1}) \ln(1+z_i) + 3\delta_n \ln(1+z) \quad (6)$$

II. $w_{de} = w_0$. In this case, we have

$$\begin{aligned} \rho_{de}(z_{n-1} < z \leq z_n) &= [\rho_{de}^0 + \frac{\delta_1}{1+w_0} \\ &\quad - \frac{1}{(1+w_0)} \sum_{i=1}^{n-1} (\delta_i - \delta_{i+1})(1+z_i)^{-3(1+w_0)}](1+z)^{3(1+w_0)} - \frac{\delta_n}{(1+w_0)} \end{aligned} \quad (7)$$

III. $w_{de} = w_0 + w_1 z/(1+z)$. In this case, $F(z)$ in Eq. (4) has the form [33, 34]

$$F(z) = (1+z)^{3(1+w_0+w_1)} \exp(-\frac{3w_1 z}{1+z}) \quad (8)$$

IV. $w_{de} = w_0 + w_1 z/(1+z)^2$. In this case $F(z)$ can be expressed as

$$F(z) = (1+z)^{3(1+w_0)} \exp(\frac{3w_1 z^2}{2(1+z)^2}) \quad (9)$$

For parameterizations III and IV, it is hard to get analytic forms of ρ_{de} as Eqs. (6) and (7). Now the Friedmann equation of the interacting models can be written as:

$$E^2(z) = \frac{H^2(z)}{H_0^2} = \Omega_r^0(1+z)^4 + \rho_m/3H_0^2 + \rho_{de}/3H_0^2 \quad (10)$$

We now fit these four models with observations. The observational data to be used are the 307 Union SNIa data [28], the Baryon Acoustic Oscillation (BAO) data from SDSS DR7 [29], the shift parameter R from WMAP5 [31], and 9 data of the Hubble parameter $H(z)$ [30]. We obtain the best-fitted parameters by minimizing

$$\chi_{tot}^2 = \tilde{\chi}_{sn}^2 + \chi_{bao}^2 + \chi_R^2 + \chi_H^2 + (h - 0.742)^2/0.036^2 \quad (11)$$

where $h = H_0/100 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. We acquire the constraints by using the MCMC method.

For 307 Union SNIa data, χ_{sn}^2 is defined as

$$\chi_{sn}^2 = \sum_i \frac{[\mu_{th}(z_i) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)} \quad (12)$$

where $\mu_{th}(z) = 5 \log_{10}[(1+z) \int_0^z dx/E(x)] + \mu_0$, and $\mu_0 = 42.384 - 5 \log_{10} h$ is a nuisance parameter. One can expand Eq. (12) as

$$\chi_{sn}^2 = A + 2\mu_0 B + \mu_0^2 C$$

where

$$\begin{aligned} A &= \sum_i \frac{[\mu_{th}(z_i; \mu_0 = 0) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}, \\ B &= \sum_i \frac{\mu_{th}(z_i; \mu_0 = 0) - \mu_{ob}(z_i)}{\sigma^2(z_i)}, \quad C = \sum_i \frac{1}{\sigma^2(z_i)} \end{aligned} \quad (13)$$

We adopt the minimization of χ_{sn}^2 with respect to μ_0 to replace χ_{sn}^2

$$\tilde{\chi}_{sn}^2 = \chi_{sn,min}^2 = A - B^2/C$$

In fact, it is equivalent to performing an uniform marginalization over μ_0 [36].

For the BAO data, one has

$$\chi_{bao}^2 = \frac{[D_V(0.35)/D_V(0.2) - 1.736]^2}{0.065^2} \quad (14)$$

where $D_V(z) = [z/H(z)(\int_0^z dx/H(x))^2]^{1/3}$.

For the shift parameter, we take

$$\chi_R^2 = \frac{(R - 1.71)^2}{0.019^2} \quad (15)$$

where $R = \sqrt{\Omega_m^0} \int_0^{z_*} \frac{dz}{E(z)}$ and $z_* = 1090$.

And for the Hubble evolution data, we have

$$\chi_H^2 = \sum_{i=1}^9 \frac{[H(z_i) - H_{ob}(z_i)]^2}{\sigma_i^2} \quad (16)$$

Note that we also have used a Gaussian prior $h = 0.742 \pm 0.036$ [37].

III. RESULTS

Now we fit our models with the observations. As the data only give very weak constraint for $z > 1.8$, we fix $\delta(1.8 < z < 1090) = 0$ in our main analysis. To obtain the constraint for a specified parameter, we marginalize over all other parameters by using the MCMC method. In addition, in all computations, we demand that ρ_{de} and ρ_m keep positive in the range of $z \in (0, 1090)$. We do not decorrelate the constraints in the different redshift bins. The constraints of δ_i are correlated. But in this way it ensures that the constraints obtained for a given bin are confined to the exact redshift range of the bin, as discussed in [28, 35]. As the fitting results might depend on the divided method of redshift bins and the dark energy models, we will study two situations:

- A. different numbers of bins with a constant w_{de} ;
- B. four different parameterization of w_{de} with a preferable division of bins.

TABLE I: The best-fitted values of δ_i in $z \in (0, 1.8)$ for 3-6 bins. The values in () is the upper boundary of the redshift bin.

Models	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6
3 bins	-0.32 (0.6)	0.12 (1.2)	5.80 (1.8)	-	-	-
4 bins	-0.17 (0.45)	-0.69 (0.9)	5.13 (1.35)	-2.54 (1.8)	-	-
5 bins	0.05 (0.36)	-1.09 (0.72)	0.16 (1.08)	10.12 (1.44)	-10.32 (1.8)	-
6 bins	0.09 (0.3)	-1.84 (0.6)	0.29 (0.9)	6.11 (1.2)	1.08 (1.5)	62.20 (1.8)

A. Effects of the number of bins

At first, to see effects of the number and locations of the redshift bins, we divide the region of $z \in (0, 1.8)$ equally into 3, 4, 5 and 6 bins respectively and assume w_{de} to be a constant. The best-fitted δ_i from the observations are shown in Table I. In the most cases the result prefers that $\delta(z)$ crosses the non-interaction line ($\delta = 0$) around $z = 0.5$ and there is likely an oscillation of the interaction term $\delta(z)$. The corresponding errors of $\delta(z)$ are shown in the Fig. 1. In 95% c.l., all results for 3-6 bins show that δ is consistent with 0 from the observations. But in 68% c.l., $\delta(z)$ is minus at the first bin ($z \lesssim 0.5$) in the 3 and 4 bins cases and at the second bin (around $z = 0.5$) in the 5 and 6 bins cases. If one uses other three parameterizations of DE that introduced in the section II, one would get similar results, i.e., there is always a bin in which $\delta(z)$ departs from 0 in 68% c.l.. As a result we may conclude from Fig. 1 that

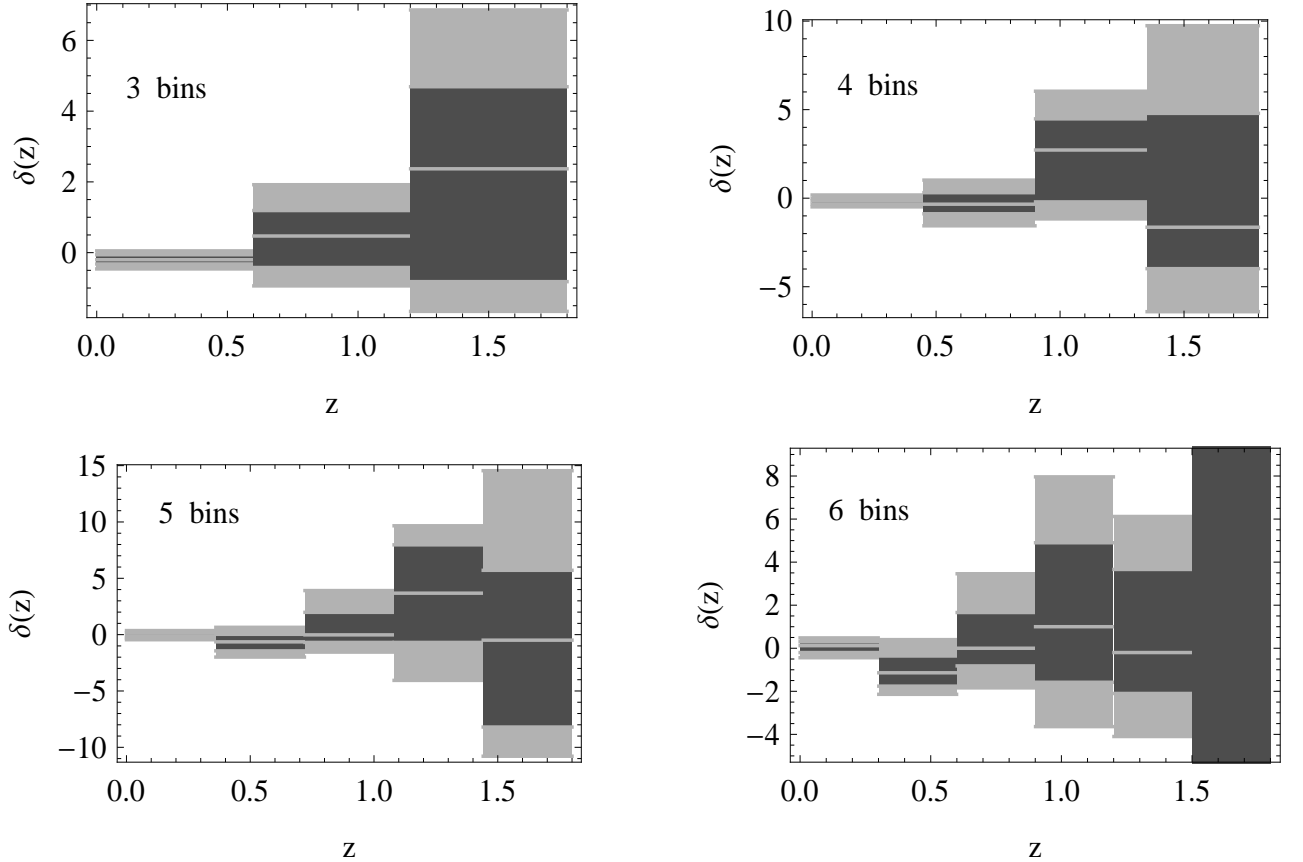


FIG. 1: Constraints of $\delta(z)$ at 68% and 95% c.l. for 3, 4, 5 and 6 bins in $z \in (0, 1.8)$. The bins are equally divided. The equation of state of DE is assumed as a constant. $\delta(z)$ is in the unit of $3H_0^2$.

1. With more bins, more finer structure of δ can be resolved, e.g., for more than 3 bins oscillation behaviors of δ appear. But for more bins the constraints of $\delta(z)$ in each bin from the observations will be weaker.

2. The errors for $z > 1$ are much bigger than that for $z \in (0, 1)$. It is mainly due to the fact that there are much less data points in the large redshift region.

It is also very likely that $\delta(z)$ crosses the $\delta = 0$ line around $z = 0.5$.

B. Effects of parameterizations of DE

By considering these conclusions, in what follows we will divide the region of $z \in (0, 1.8)$ into three bins as: $(z_0 = 0, z_1 = 0.2, z_2 = 0.5, z_3 = 1.8)$ [27], which is the case adopted by most discussions in the literature and from which fine constraints of $\delta(z)$ could be obtained indeed. Four parameterizations of DE introduced in section II will be used. The best-fitted parameters and the constraints at 68% and 95% c.l. are shown in Table II. The best-fitted

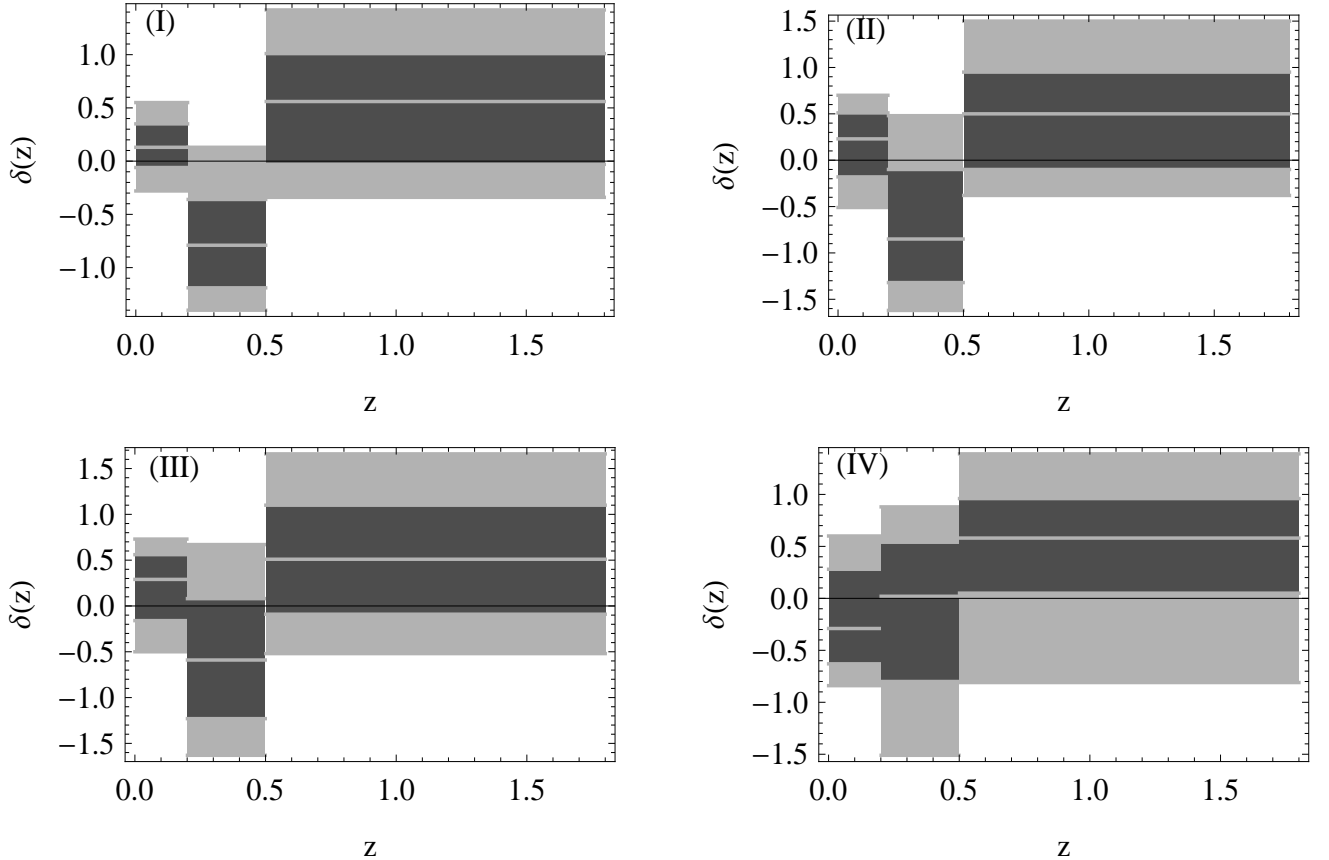


FIG. 2: The constraints of δ_i at 68% and 95% c.l. . δ_i is in the unit of $3H_0^2$.

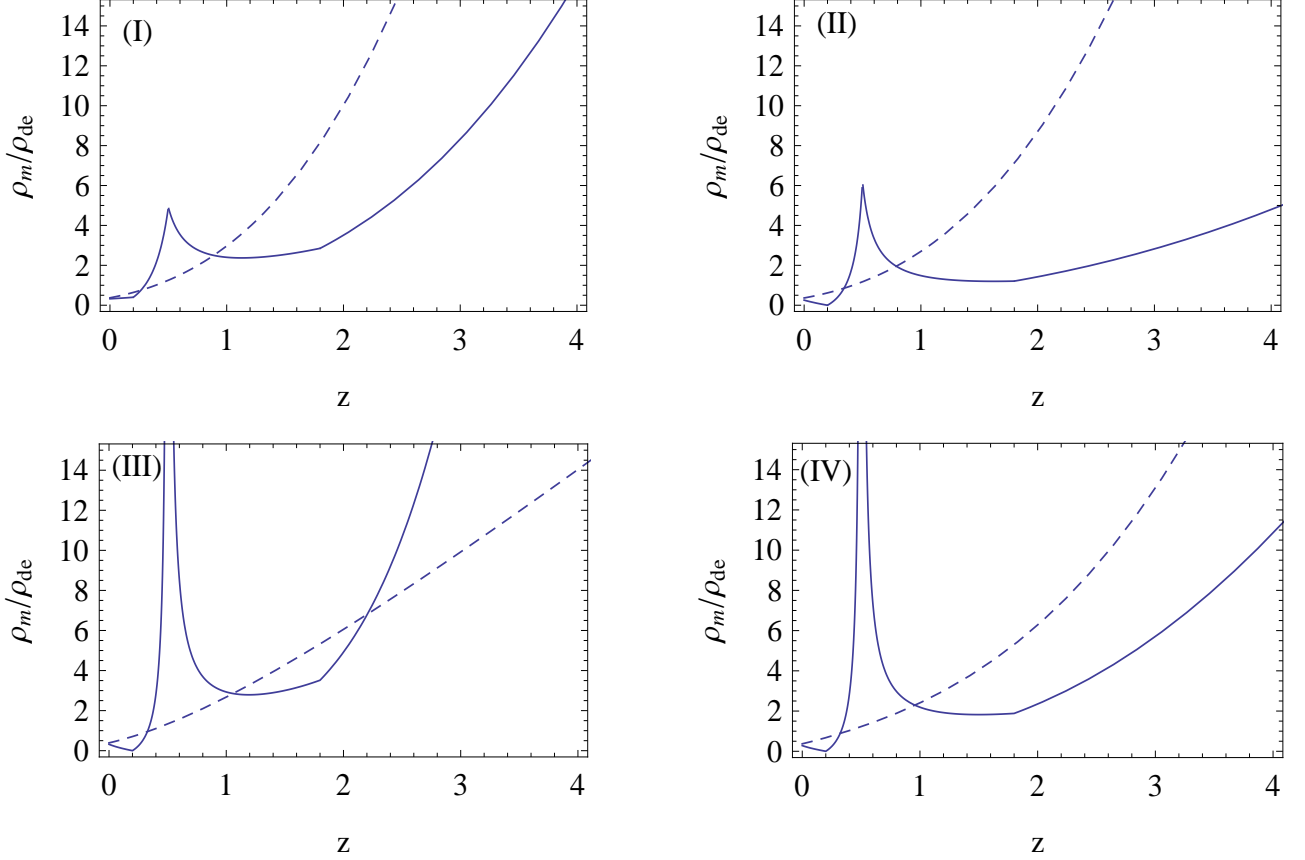


FIG. 3: The behaviors of $r = \rho_m/\rho_{de}$. Solid curves are for the best-fitted models shown in Table II (with $\delta_4 = 0$), while the dashed curves for the best-fitted models with $\delta(z) = 0$.

parameters for models with δ_4 unfixed are also shown in Table II. There are almost no differences between the models with δ_4 fixed and unfixed. The corresponding 68% and 95% constraints are shown in Fig. 2, and the Fig. 3 shows the behaviors of the ratio $r = \rho_m/\rho_{de}$ in the best-fitted models with interaction, compared with the cases without interaction.

I. $w_{de} = -1$

From Table II and Fig. 2, it is obvious that δ_2 is negative, while δ_1 and δ_3 are positive, which implies that $\delta(z)$ could cross the $\delta = 0$ line at a recent time and the decay direction between DM and DE could be variable. When $\delta(z)$ crosses the non-interacting line, the effective equation of state of DE will cross the cosmological constant ($w = -1$) line. Thus DE behaves as a quintom fluid. As shown from Table II and Fig. 2, δ_2 departs from 0 beyond 68% c.l. But models (III) and (IV), which have more degrees of freedom, show that it is still consistent with $\delta = 0$ everywhere in 68% c.l. . In addition, it can be seen from Fig. 3 that the interaction can help to alleviate the coincidence problem in this case.

II. $w_{de} = w_0$

For this parametrization, the situation is similar to the case of $w_{de} = -1$. The sign of $\delta(z)$ can be varied in the different bins. The possibility of $\delta_2 < 0$ is larger than 68%, but less than 95%. The coincidence problem is also alleviated in the best-fitted model.

III. $w_{de} = w_0 + w_1 z / (1 + z)$

In this case, there is still a downward departure of δ_2 from 0, but now the constraint is consistent with $\delta(z) = 0$ in 68% c.l. It looks from Fig. 3 that the coincidence problem could not be alleviated in this case. Note that to avoid a serious degeneracy, we have assumed a prior $\Omega_m < 0.37$ here.

IV. $w_{de} = w_0 + w_1 z / (1 + z)^2$

In this case, $\delta(z) = 0$ is consistent with the observations in 68% c.l. There is still a possibility of crossing the non-interacting line. The coincidence problem can be alleviated.

As expected, the resulting constraints are effected by parameterizations of w_{de} and divisions of bins. But for all cases we have considered here we see that the interacting term prefers to have a behavior crossing the non-interacting line, and there might exist an oscillation $\delta(z)$ in the most cases.

IV. CONCLUSION

We have investigated the constraints of the interaction between DE and DM from the observational data. To make the constraints independent of specific interacting forms, we divide the whole redshift into four bins. In each bin $\delta(z)$ is set to be a constant δ_i . First we have estimated effects of numbers of bins by consider 3-6 bins with a constant w_{de} , from which we get some common features and choose a preferable division of bins: ($z_0 = 0, z_1 = 0.2, z_2 = 0.5, z_3 = 1.8$). For models of DE, we have adopted four parameterizations of w_{de} . The resulting constraints of δ_i depend on these parameterizations. But there are also some common features of the interaction. The results are summarized as follows.

1. The observational data prefer that $\delta(z)$ crosses the $\delta = 0$ line and has an oscillation behavior at a recent time, which implies that the decay direction can be variable. It is similar to the case that the equation of state of DE is likely to cross the cosmological constant ($w = -1$) line. For many well studied phenomenological interacting forms, such as $\delta = \lambda \rho_{dm}$ and $\delta = \lambda(\rho_{dm} + \rho_{de})$, the sign of $\delta(z)$ is unchangeable. Our results raise the

possibility that $\delta(z)$ can have different signs at the different times. It implies that more general phenomenological forms of the interaction should be considered, if the interaction indeed exists.

2. The constraints given from observations show a departure of $\delta(z)$ from 0 beyond 68% c.l. for the $w_{de} = -1$ and $w_{de} = w_0$ parameterizations. But for other two parameterizations of DE, $w_{de} = w_0 + w_1 z/(1+z)$ and $w_{de} = w_0 + w_1 z/(1+z)^2$ which have more degrees of freedom, the constraints are consistent with $\delta(z) = 0$. To confirm the existence of the interaction, more observations and theoretical studies are needed.

3. The coincidence problem can be alleviated in the three cases of four models, compared to corresponding ones without the interaction. The ratio $r = \rho_m/\rho_{de}$ will evolve more rapidly (slowly) when $\delta(z) < 0$ ($\delta(z) > 0$) than the cases without the interaction. The decay of DE to DM ($\delta(z) > 0$) can alleviate the coincidence problem, while the decay of DM to DE ($\delta(z) < 0$) will make it more severe. Though $\delta(z)$ is negative somewhere in the best-fitted models, its effects can be offset by that of the $\delta(z) > 0$ regions and the period of $r \sim O(1)$ can be longer than that of the corresponding non-interacting models. This way the coincidence problem is alleviated.

Due to the ignorance on the properties of DE and DM, to study the dark sector interactions one always needs to assume some models of DE and DM. DM is always assumed as pressureless fluid, while there exist plenty variants of DE models. In the sense of phenomenology, the main difference among those models of DE is just the number of parameters. In our discussions, we have adopted four widely used parameterizations for the equation of state of DE in the literature. For different numbers of bins, there are also common results. Therefore our results are of some universality in some sense that $\delta(z)$ prefers to cross the $\delta = 0$ line and have an oscillation behavior. In particular, this results indicate that if there does not exist any interaction between DM and DE, the model of DE should be paid special attention with an oscillating equation of state, because the oscillating behavior of the interacting form is mathematically equivalent to the case without interaction, but with an oscillating equation of state of DE.

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TABLE II: Constraints of the interaction between DE and DM. For each parametrization of DE, the first line refers to positions of Maximum Likelihoods and errors of parameters at 68% and 95% c.l. , the second line gives the best-fitted values of the parameters, in which we have set $\delta(z > 1.8) = \delta_4 = 0$. The third line is for the best-fitted models with δ_4 unfixed. The value in $\{\}$ means this parameter has been fixed.

EoS of DE		h	Ω_m^0	w_0	w_1	δ_1	δ_2	δ_3	δ_4
I. $w = -1$ ($\delta_4 = 0$)	ML	$0.73_{-0.02-0.06}^{+0.02+0.05}$	$0.28_{-0.06-0.12}^{+0.04+0.08}$	$\{-1\}$	$\{0\}$	$0.13_{-0.19-0.41}^{+0.22+0.42}$	$-0.79_{-0.40-0.61}^{+0.43+0.92}$	$0.56_{-0.59-0.90}^{+0.45+0.86}$	$\{0\}$
	best-fitted	0.734	0.244	$\{-1\}$	$\{0\}$	0.127	-0.804	0.770	$\{0\}$
δ_4 unfixed	best-fitted	0.735	0.246	$\{-1\}$	$\{0\}$	0.133	-0.810	0.778	-0.097
II. $w = w_0$ ($\delta_4 = 0$)	ML	$0.73_{-0.02-0.05}^{+0.03+0.06}$	$0.28_{-0.05-0.13}^{+0.04+0.08}$	$-0.86_{-0.28-0.72}^{+0.11+0.18}$	$\{0\}$	$0.23_{-0.41-0.74}^{+0.28+0.47}$	$-0.85_{-0.47-0.77}^{+0.75+1.33}$	$0.50_{-0.60-0.88}^{+0.45+1.00}$	$\{0\}$
	best-fitted	0.735	0.204	-0.792	$\{0\}$	0.484	-1.545	1.107	$\{0\}$
δ_4 unfixed	best-fitted	0.735	0.218	-0.797	$\{0\}$	0.518	-1.558	1.128	-0.603
III. ($\delta_4 = 0$) $w = w_0 + w_1 z / (1 + z)$	ML	$0.73_{-0.02-0.05}^{+0.03+0.05}$	$0.29_{-0.06-0.11}^{+0.05+0.08}$	$-0.97_{-0.29-0.75}^{+0.22+0.38}$	$1.22_{-1.81-2.75}^{+0.37+0.75}$	$0.29_{-0.45-0.79}^{+0.27+0.44}$	$-0.59_{-0.64-1.04}^{+0.67+1.26}$	$0.51_{-0.60-1.03}^{+0.59+1.15}$	$\{0\}$
	best-fitted	0.733	0.242	-0.632	-1.507	0.574	-1.802	1.134	$\{0\}$
δ_4 unfixed	best-fitted	0.734	0.232	-0.627	-1.533	0.551	-1.801	1.131	-0.388
IV. ($\delta_4 = 0$) $w = w_0 + w_1 z / (1 + z)^2$	ML	$0.73_{-0.02-0.06}^{+0.02+0.05}$	$0.30_{-0.05-0.14}^{+0.04+0.08}$	$-0.98_{-0.42-0.94}^{+0.65+1.67}$	$2.9_{-12.2-38.3}^{+2.0+4.1}$	$-0.29_{-0.34-0.55}^{+0.57+0.89}$	$0.02_{-0.82-1.53}^{+0.52+0.86}$	$0.58_{-0.53-1.39}^{+0.38+0.81}$	$\{0\}$
	best-fitted	0.733	0.221	-0.594	-2.115	0.524	-1.807	1.288	$\{0\}$
δ_4 unfixed	best-fitted	0.733	0.228	-0.598	-2.098	0.540	-1.805	1.287	-0.277